2013

Mathematics Interns at Holt High School

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[LESSON STUDY REPORT]

Lesson: Interpreting Parametric Graphs

Part 1: Background Information & Research Goals

Background information about our school:

As a cohort of math interns at Holt High School, we organized a total of four case study lessons throughout our student teaching year. Since the content that each of our team members' taught in their placements was different and ranged from geometry, algebra I, algebra II, Pre-calc, remedial math concepts, we tried to have our four lesson topics alternate among the collection. Our topic of focus for this lesson study fits within the Algebra II and Pre-calc category.

The mathematics department at Holt Senior High school does not use a mathematics textbook. As a result, the department collaborates extensively. They work together to develop course and unit objectives and daily lesson tasks that align to the common core state standards.

Broad Goals for Students:

Some of the broad goals that the math department at Holt share are for students:

- To preserver through challenging tasks.
- To adequately display an understanding of concepts through writing.
- To build students ability to reason abstractly and quantitatively.

Additional Research Goals:

Mathematics is not a favorite subject among some of our students at Holt schools which can lead to disengagement and apathy in our courses. Therefore, our team always looks for ways in which we can ensure students stay motivated to learn. Our additional goals for our lesson study are:

- To increase student engagement in the lesson by relating the task to a context applicable to real life.
- To diversify the number of students participating by cold calling.

Part 2: Mathematics Overview

Topic and Understanding Goals of the Research Lesson: Interpreting Parametric Graphs

As a team we like to decide on lesson study topics that students have most trouble grasping. Since we all have little experience with the Holt curriculum, having never taught it before, we sought out our experienced mentor teachers to help determine what concepts are hard for students to understand. In general students tend to struggle with interpreting graphs. When dealing with parametric situations, where we can graph two dependent variables against one another, it seems to be even more confusing for them. For example, any object in projectile motion can have its change in height and velocity over time modeled by the following equations: $h\ \ t\ = -\frac{1}{2} \cdot g \cdot t^2 + v_y \cdot t + y_0$ & $v\ \ t\ = -g \cdot t + v_y$. When height and velocity are graphed against one another ($h\ \ t\ \ vs\ v(t)$), students have trouble seeing the graph as relating the object's height to its velocity but instead interpret the graph as the object's actual path through the air. Therefore, as part of our lesson study we wanted students to not only graph height verses velocity but velocity verses height. This would help students to think twice about the possibility of the graph being the object's actual motion. This move would also allow the class to discuss ideas like why the equations can be graphed both ways and if the graphs both represent functions. To guide students into the next topic in the unit, composition of functions, we wanted students to begin to make connections between the graphs and the equations used to make the graphs. We wanted students to see that since both equations ($h(t)$ & $v(t)$) are dependent on the same variable time, the equations could be manipulated so that height is dependent on velocity ($h(v)$) or velocity is dependent on height ($v(h)$).

Understanding Goals:

From our lesson we want students to be able to accurately interpret how an object's change in height relates to its change in velocity and vice versa through analyzing graphs of the two dependent variables graphed against one another.

We also wanted students to make connections between the graphs and the equations by considering possible ways to compose a single equation containing the two variables, height & velocity, which would model what is happening with the graphs.

Summary of the Mathematical Context and Rationale for Lesson Design:

This lesson begins a unit involving parametric equations. Students were first exposed to parametric equations in the previous unit where they developed a foundational understanding of the need for and use of parametric equations. This earlier unit primarily focused on helping students understand that parametric equations are used to model the variety of different types of motion an object could travel in (linear motion with constant velocity, projectile motion, and circular motion). Throughout the unit students focus on developing an understanding for the variables in each equation in order to accurately use the equations to answer questions such as "how fast is the object traveling when it is at a height of 10 feet?"

In this new unit, students will transition from using the equations to solve situations to analyzing the equations to find X & Y – intercepts and domain and range. In order to do this students will need to have an understanding of how the parameter (usually time) will cause restrictions of the domain and range for graphs that involve just the dependent variables (for instance height and velocity). Therefore, the goal of this lesson was to push students beyond relating height and velocity to the independent variable, time, and to start considering what happens if we compare how the height and velocity are changing in regards to one another. Using the graphs of height vs. velocity and velocity vs. height, students were asked to compare characteristics of these graphs and use them to solve situations. This will eventually lead students to develop a rule that relates the height of an object in projectile motion in terms of velocity and a rule that relates velocity in terms of height. In cases when we are not given a time but instead are interested in finding the velocity of an object when it reaches a certain height or the objects height when it is traveling at a specific velocity, students will see the benefit of having a height equation in terms of velocity or a velocity equation in terms of height.

Part 3: Instructional Plan

Lesson	Students are working *(details about how students are configured, what work they are doing and how they are recording their work)*	Anticipated Student Thinking/Questions	Teacher Moves
Launch (15 minutes) 1. (2.5 minutes) Take out some juggling balls and juggle. Ask the class what is changing about the juggling balls as time goes on. 2. Introduce situations**:** Marty the Juggler is Juggling bowling pins at the circus. He throws each pin in the air from a height of 4.5 ft and with a vertical velocity of 24 ft/s. 3. As a class, develop a set of parametric equations that model the bowling pin's velocity & height as a function of time. 4. Pose this question to the class: "What height will each bowling pin be at when it is traveling only 5 ft/s?" Then ask for the process students would usually go through to solve the situation. Students would usually have a two step process; plug 5 into velocity equation & solve for t	1. Students will be watching the video clip, following along, sitting in their assigned seats. 2. Students are solving the problem given to them on the Juggling Circus Pins Worksheet. They will be working on this problem individually. 3. Students will be volunteering their thoughts to the teacher about the set of equations that model the velocity and height of the bowling pin. The students will raise their hands and participate when they have ideas about the potential parametric equations. 4. Students will be focusing to find the height of the pin when the pin is traveling at 5ft/s. They will be using the two step process by plugging 5 into the velocity equation and solving	1. 2. Students may understand what it means to throw something in the air with a vertical velocity, but relating to this mathematically may be a struggle. 3. Students may be overwhelmed by the number of variables in the equations. They might not realize that most of them remain constant throughout the situation and only time varies. This applies to both equations. 4. Students may be confused with what a negative velocity implies (downward motion). They may just change velocities to being positive and in some situations this may affect outcomes. 5. This idea may make sense intuitively when thought about in a real world sense but some students may not make the connections to what this means mathematically.	1. Teacher will try to get students to think about what is changing about the juggling balls. Height, velocity, & horizontal distance should all come up but teacher will explain to students that today we are only going to focus on the height & velocity change of the juggling balls. 2. Teacher will introduce questions for students. 3. Teacher will ask for equations from students. Teacher will ask what variables to take into consideration. What variables remain constant? What variables are not constant? 4. Show how negative velocity is downward motion. Show how to plug in 5 and solve for time and then plug in time to find the height. Ask why it is important that we use a two-step process? Why is the

& then plug t into the height equation to find height. 5. Ask the class if they think there is a way to avoid the two step process. If height is dependent on time & velocity is dependent on time, could height be dependent on velocity or velocity dependent on height? Tell the class that today we are going to begin to explore that idea. **<u>Explore (35 minutes)</u>** **1.** Hand out the task to the students. Students will be first asked to independently complete the tables that are on the back side of the handout. **2.** Students will then use that information to plot points on the graph provided to them. There will be 11 pink papers that will have students graphing velocity on the x-axis & height on the y-axis. There is another 11 yellow papers that has students graphing height on the x-axis and velocity on the y-axis. 3. Students will then answer the following questions by	for t, then plugging that into the height equation to find the height. Then they will be sharing out their solutions. 5. Students will start to think of other ways to solve this step 4 without using the two step method. Students will share ideas to the teacher of methods they could use to avoid using the two step method. 1. Students will complete the tables that are on the handout individually 2. Students will be taking the points from the table and plotting them onto the graph on the same worksheet. They will either be plotting the relationship between velocity and height or the relationship between the height and velocity (same components on different axes) 3.a - c. Students will be working on the problems individually. 4. Students will be	1. 2. Students, being so used to time being the dependent variable, may plot H vs. T or V vs. T. Having an "independent variable" on both the x and y axis may be confusing to them. 3a. Students might ask to be reminded about what x-intercepts are. Without time on the graph students may be unsure how to find it to describe when the event (root) occurs in the real life situation. Student might struggle relating back to x(t) and y(t) from only the x-y graph. 3b. Same thing here. Students might be unsure how to describe "what this point means" without having seeing time on the graph. Describing the event through only height and velocity might seem vague to them. c.) Students will question why velocity is negative at this point. "What does it mean to have a negative velocity?" 4a) Students should notice that the graphs axis are flipped but might not see that that makes them inverses. Students might	velocity negative? What equation do we use? How would not paying attention to the negative change the outcome? 5. What happens if we did not use a two-step process? If two different variables are dependent on the same variable, what can we do? How can we explore this idea? 1. Go over the worksheet and explain that each student is to complete the tables. Ask if there are any immediate questions about the table? 2. Which variable is dependent? Which variable is independent? Where is the dependent variable graphed? Independent? Why are there two independent variables? How does this occur? What does your pink/yellow paper tell you to graph? 3a. What is an x-intercept? Is time apparent on your graphs? Where is time, how can the two objects happen without time? With velocity and height what type of relationship is created? b. What does it mean when I give you a

themselves: a. On your graphs, identify the x-intercept. Explain what this point means in terms of the variables in this situation. b. On your graph, go to an x-value of 12. Find the corresponding y-value. Explain what this coordinate means in terms of the variables in the situation. c. On your graph, go to the point that corresponds to Marty catching the bowling pin at a height of 4 feet. Mark this point. **4.** At this point students will find a partner who has a different color worksheet and answer the remaining questions about the two graphs: a. How are your graphs similar? How are they different? What do the similarities or differences tell you about how velocity & height can be related? b. Is there a way we can develop an equation that relates velocity and height together? **<u>Summarize/Share and Discuss (10 minutes)</u>** **1.** Ask the class what they noticed about the two graphs.	pairing up with another student based of the color of the sheet of paper. The students will be working together at desks, discussing questions a -c. 1. Students will be participating in a group discussion. They will be answering questions that the teacher will be asking about the questions they answered with their partner. 2. Students will be sharing their ideas about the equation that helps relate velocity and height.	question why one graph is a function and the other is not. Also students might see that both graphs have the same variables and consider them "the same" not realizing that this has implications, namely the function/not a function aspect of it. 4b) They are familiar with quadratics and might try to fit both graphs to a quadratic rule. Some might be able to find the rule for the parabola and then realize that they other is an inverse and make the inverse equation that way. Although this strategy works it will mean they were able to complete the task without making an f(g(x)) which means they may be confused with the purpose of the activity. It might seem like review. Students might ask how to invert a quadratic since they have not done it consistently since last year. 1. Students should realize that the height and velocity are graphed on opposite axis making the two graphs inverses. 2. Students, losing track of the input and output of each rule, may just plug one rule into the other essentially making a v(h(t)) or h(v(t)). Students might ask how to invert a	value? Does the graph help represent this relationship? How does this relate to the variables? c. How can velocity be negative if it is not slowing down? How do we find point A on the yellow/pink paper? 4a. What did you notice between papers? What specifically is different on the graphs? Is this an inverse? How can this be graphed with the same variables but put on different axis? How does this function work? b. What do you need to make an equation? Will your equations be the same? Is this a quadratic? How did you create your rule? Does it make sense the order you multiply in? In the launch how did it make sense? 1. What did you and your partner notice about the two graphs? Are the two graphs inverses? 2. From these graphs we see that we can create an height equation in terms of velocity (h(v)) and a velocity equation in terms of v (v(h)). First of all, what are the benefits of doing this? What are some ideas in how these equations

<table>
<tr><td>Asks students what was similar & different about the graphs.
2. Let students share any ideas they had about an equation that relates velocity & height together so that we can bypass having to always find time.</td><td></td><td>quadratic or linear. This old misconception (or lack of retained understanding) may hold them back.</td><td>look?

If students do not have many ideas push students to think about writing the height and velocity equations so that they solve for time. Students have height and velocity in terms of time. What is time in terms of height or velocity? How might knowing this help?</td></tr>
<tr><td colspan="4">Exit Slip: Find a solution to the following problem that does not involve finding time first. If Marty throws his pin with the same vertical velocity as before, how fast will the pin be traveling when he catches the pin at a height of 4.2 ft?

Note: Urge students to use the equations to answer the above questions. Students may estimate an answer using their graph if they like but they also need to find an answer using their equations.</td></tr>
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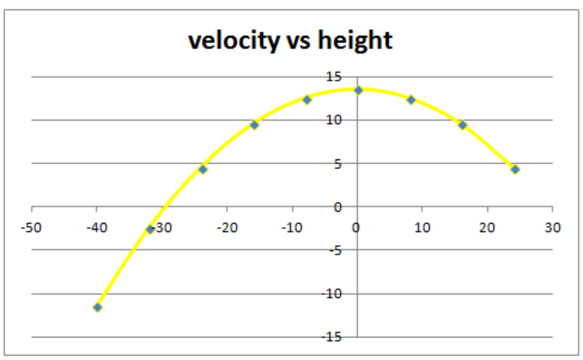
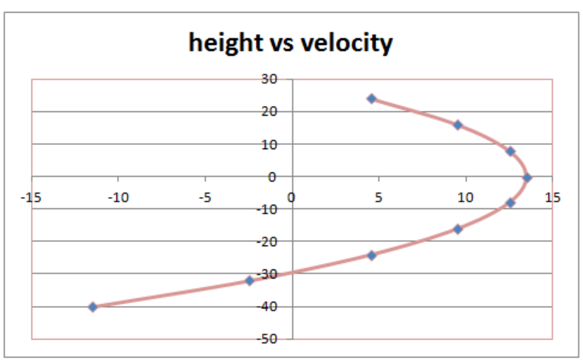
Overview of student solutions:





Time	Height	Velocity
0	4.5	24
0.25	9.5	16
0.5	12.5	8
0.75	13.5	0
1	12.5	-8
1.25	9.5	-16
1.5	4.5	-24
1.75	-2.5	-32
2	-11.5	-40

Part 4: Observation guide

Observation Questions

Based on our goals and research questions, we selected the following questions for observers to consider while the lesson was enacted:

1. Are student's conversations centered on exploring what information can be taken from each graph?
2. Are student's expressing an understanding of the similarities/differences between the two graphs?
3. Are student's persevering in thinking about the purpose of the composition of the height and velocity equations and what it does?
4. How are students reacting to cold calling during whole group discussion?
5. Are students overcoming the common misconception that the shape of the graph represents the shape of the path of the object?

Additional Observer Information

When observing, keep in mind that the class will be very spread out. There will be 3-4 groups in the classroom and 2-3 groups in the hallway or in the empty classroom next door. Make sure to give the groups outside of the classroom equal attention and to take note of what these groups are doing. This class works spread out in small groups very often so it is nothing new to the students for teachers to walk out and talk to them while they are working. Also, make sure to take notes as you observe and please allow the lesson to proceed naturally, staying as quiet as possible.

Notes from Observers

While observing during small group discussion, most groups seemed to pick one graph and focus on it to answer questions. Most groups used the one with velocity on the x-axis and height on the y-axis. We came to the conclusion that they used this one more often because its shape (quadratic) was more familiar to them. Also the other graph was not a function which meant multiple outputs depending on what x you chose. We didn't notice them using the other graph as a way to cross reference their conclusions at all. We weren't sure what this meant but it could have meant that they only understood the relationship portrayed in one graph and thus only used the one they understood or that they didn't see the graphs as portraying the same relationship so the thought of cross referencing never occurred.

Another common observation was that groups, especially during small group discussion, thought that certain students were struggling to see that the graph was not representative of the path of the object, but instead showed the relationship between velocity and height. The arch of a tossed object is parabolic so this is understandable, but we wanted students to pay more attention to the relationship between the variables and not the similarity between the shape of the graph and the path of the object.

Part 5: Post lesson Analysis

Key points raised at the Post-lesson discussion

- Timing of the task was different between the two hours.
  - One hour was able to compare what the graph looked like, but we were not sure if they understood the graphing.
  - The next hour did not complete the worksheet possibly due to not understanding the purpose of the activity.
- Grouping of students affecting the way students worked with each other and interacting with the whole group conversations.
- During the summary of the first hour we noticed that very few students were responding, we thought it would help to cold call other students to help enrich the discussion.
- Students were not able to understand what the graph was representing and what forces took place in order to create such a graph shape. We should have discussed more during the juggling what all of the factors may be that are acting onto the ball's path.
- Due to lack of discussion and students not finishing the worksheet, we believe many students seemed to be off task and lacked perseverance.

Team summary of findings

During our post lesson discussion, we talked about the differences between the first and second time we enacted the lesson. We found that in 2nd hour there was more of a discussion while students were completing the worksheet, most finished the worksheet and they got to compare graphs in a discussion. In 3rd hour, the students had more questions about the task, did not finish the worksheet and did not have a chance to discuss their work.

For both hours we noticed that Danae chose not to move students into different groups although some people were absent, which made it difficult because there were different amounts of people in different groups. We thought that it may have benefited the lesson to move the students all to one side of the classroom since there were many absences and this may have helped them work better with their peers. We also noticed that although Danae did use cold calling more than usual, there could have been more cold calls so more students could be engaged in the group discussion. In the first class, only three different students were volunteering and Danae cold called on a few more students. The group talked about how only having a handful of students compared to the whole class discussing the worksheet that many will not feel connected to the material and become off task or have misconceptions that we were not able to address. We also discussed the ways students were off task during the lesson and how she could get students back on task, in hopes that the students would have made more progress and to promote discussion to help build student understanding.

Since the students were lacking in the discussion, as a group we were not convinced that students were able to understand their own graphs let alone a partner's graph. We decided that we should have talked about in the launch what it means for the ball to be thrown in the air and to land in the opposite hand when juggling. We also thought if the students were able to explain the same idea with the specific equation given then they truly would have mathematized the situation and shown and overall understanding of the lesson. Overall, we were not convinced that the lesson was too challenging for the students but that there was a lack in perseverance and the ideas that needed to be addressed were not questioned by the student. As a group, we decided as the teacher we need to keep students on task in order to promote discussion to reveal misconceptions necessary to help all students develop their own type of understanding.